

Virial and Energy Dissipation in Measurement of Dynamic Acoustic Forces Using Bimodal-frequency Excitation of Micro-cantilever Array

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Abstract

Virial and energy dissipation, related to oscillation observable responses, possess complementary information regarding acoustic force measurements. In this paper, we introduce a mathematical framework describing the analytic relationship between oscillation observables and energy quantities at the second eigenmode in the measurement of dynamic acoustic forces. We utilize a bimodal-frequency excitation scheme for actuation of the micro-cantilever array to obtain high-sensitivity frequency bands. Herein, we analyze the virials of acoustic force interaction and the energy dissipation levels on the domain of acoustic force frequency. For our case, we obtain the high-frequency bands of around 200-270 kHz and 440-570 kHz for the force strengths in the range of 4.0-36.0 pN. In addition, results of virials and dissipated power with respect to acoustic force strengths are introduced for low- and high-sensitivity frequency regions. Therefore, the energy quantities can be robustly utilized to determine high-sensitivity frequency windows in the measurement of dynamic acoustic forces.

Key words: Virial, energy dissipation, dynamic acoustic force measurement, bimodal-frequency excitation, micro-cantilever array

1. Introduction

Acoustic emissions are detected precisely by the transducers, manufactured using Micro-Electro-Mechanical Systems (MEMS) technologies. The conditions of structures such as gas pipes and storage tanks under dynamic loads are identified by characterizing acoustic signals [1]. In addition, failure mechanisms of different materials generate elastic acoustic waves with various amplitudes and frequencies. For instance, acoustic emissions at the frequencies in the range of 90-310 kHz are emitted owing to the matrix cracking, debonding, and pull-out of carbon/epoxy material [2]. Thus, needs for characterization of acoustic signals based on frequency bands leads to in-depth investigations on high-sensitivity measurement techniques. Accordingly, fluctuations in energy dissipation observed in the measurement of acoustic emissions can be used to determine high-sensitivity frequency windows.

In AFM operations, energy dissipation processes owing to tip-sample interaction force are described using virials and dissipated power at eigenmodes of micro-cantilevers. In tapping-mode scanning force microscopy, phase shift is related to energy dissipation for low-quality factors using analytical expressions [3]. In another work, a theory of phase contrast is presented in consideration of energy dissipation in multi-frequency AFM [4]. Besides, dissipated energy on the sample surface

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is determined as a function of amplitude in amplitude modulation AFM [5]. In this current work, variations in virials and power dissipation at higher mode with respect to acoustic force strength and frequency are explored for the measurement of dynamic acoustic forces.

Sensitivities to tip-sample interaction forces are explored in the several works dealing with responses of the AFM micro-cantilevers under multi-frequency excitations [6-9]. Multimodal operations can also be used to enhance the sensitivities of oscillation observables to acoustic forces. Accordingly, it is remarked in our earlier work that phase sensitivity to static acoustic force at first eigenmode is improved by applying driving forces under a bimodal-frequency excitation scheme [10]. In this current work, the micro-cantilever array is driven under bimodal-frequency excitation to increase the width of the high-sensitivity frequency band in the measurement of dynamic acoustic forces. In the following works, micro-cantilever array based acoustic transducers are used to detect acoustic emissions within diverse frequency bands. Acoustic emissions are measured by utilizing micro-cantilevers with different resonant frequencies in consideration of air damping [11]. In another work, a multi-channel piezoelectric acoustic transducer based on a bimorph micro-cantilever array is developed to operate in a viscous fluid environment [12]. Moreover, signal processing functions of basilar membranes in the audible frequency band are imitated using piezoelectric micro-cantilevers [13].

In our earlier work, sensitivities of oscillation observables and energy quantities at the first two eigenmodes to static acoustic forces were investigated for single- and bimodal-frequency excitation schemes [10]. In this study, the micro-cantilever array is excited using a bimodal operation scheme to achieve high sensitivities to dynamic acoustic forces. Deflections at second eigenmode are determined numerically and energy quantities are calculated analytically, utilizing amplitude and phase shifts. Results of dissipated power and virial as a function of acoustic force frequency are introduced for diverse force strengths. Moreover, variations in virial and dissipated power with respect to acoustic force strength are analyzed for low- and high-sensitivity frequency regions. Therefore, the results of the energy quantities are evaluated in the light of virial theorem and energy conservation principle.

2. Mathematical Framework

The bimodal-frequency excitation model, the energy quantities and the details of numerical computation are introduced and described in this section.

2.1. Bimodal-frequency excitation model

We determine the deflections of the micro-cantilevers undergoing dynamic acoustic forces by solving the Equation Of Motion (EOM) (Eq. (1)) numerically.

$$m_e \ddot{z}_2(t) = -k_2 z_2(t) - \frac{m_e \omega_2}{Q_2} \dot{z}_2(t) + F_{exc,1} + F_{exc,2} + F_{Acoustic} \quad (1)$$

In the bimodal operation scheme, the micro-cantilevers are driven with the external excitation forces at the first and second eigenmode frequencies [10]. In the forced oscillator harmonic model, a micro-cantilever in the array is regarded as a point-mass model. In Eq. (1), z_2 , k_2 , ω_2 , and Q_2 are the instantaneous deflection, the stiffness, the angular resonance frequency and the quality factor for the second eigenmode respectively. Micro-cantilever deflections under the damping effect of the air as an operation environment are represented by the quality factor [11]. $F_{exc,1}$ and $F_{exc,2}$ are the external excitation forces with the force strengths F_1 and F_2 at the first ($\omega_1=2\pi f_1$) and the second ($\omega_2=2\pi f_2$) angular resonance frequencies respectively (Eq. (2, 3)).

$$F_{exc,1} = F_1 \cos(\omega_1 t) \quad (2)$$

$$F_{exc,2} = F_2 \cos(\omega_2 t) \quad (3)$$

Driving the micro-cantilevers at resonance frequencies under bimodal-frequency excitation, we obtain oscillations of the resonant micro-cantilevers. Dynamic acoustic force $F_{Acoustic}$ is modeled as a cosine signal with the force strength F_{Ac} at angular frequency ($\omega_{Ac}=2\pi f_{Ac}$) as introduced below.

$$F_{Acoustic} = F_{Ac} \cos(\omega_{Ac} t) \quad (4)$$

Deflections of the resonant micro-cantilevers under acoustic forces at different frequencies f_{Ac} are determined for the second eigenmode. A micro-cantilever deflection is approximated as follows [14].

$$z_2(t) \approx A_2 \cos(\omega_2 t - \phi_2) + H(\epsilon) \quad (5)$$

In Eq. (5), A_2 is the amplitude, and ϕ_2 is the phase shift at the second eigenmode with respect to free oscillation in the absence of acoustic force. $H(\epsilon)$ term represents contributions of acoustic force frequencies and higher eigenmodes. Influences of dynamic acoustic forces on the periods of free oscillations are ignored due to resonant oscillations of the micro-cantilevers.

2.2. Virial and dissipated power

Energy dissipation occurring in the measurement of dynamic acoustic forces can be described using the energy quantities such as virial and dissipated power. Observables of free oscillations of the micro-cantilevers vary considerably due to interaction with acoustic forces. The analytical expressions of virial and dissipated power for the eigenmode i are introduced for acoustic force measurement in our earlier work [10]. For this case, i is equal to 2 since we determine the results of the energy quantities for the second eigenmode. The energy storage process, occurring in the presence of external excitation force and acoustic force, is represented by virial term, whose expression is introduced as follows.

$$V_2 = -\frac{1}{2} F_2 A_2 \cos(\phi_2) \quad (6)$$

Mechanical energy supplied by external excitation force is lost due to the dissipative interaction with the operation environment and acoustic forces. Based on virial theorem and energy conservation principle, the analytical expression of dissipated power for the second eigenmode is given below [3, 10].

$$P_2 = -\frac{1}{2}F_2A_2\omega_2 \sin(\phi_2) - \frac{1}{2Q_2}\omega_2(A_2)^2k_2 \quad (7)$$

Dissipated power term is used to describe energy dissipation, considering different periods of oscillations [15].

2.3. Numerical simulation

Instantaneous deflections at the second eigenmode are obtained by solving the EOM (Eq. (1)) numerically using the Fourth order Runge-Kutta method. Coefficients of the EOM and dimensional parameters of three AFM micro-cantilevers in the array are introduced in Table 1 as in [16].

Table 1. Specification of micro-cantilever array

Micro-cantilever	l(μm)	w(μm)	t(μm)	k ₁ (N/m)	k ₂ (N/m)	Q ₁	Q ₂	f ₁ (kHz)	f ₂ (kHz)
Bruker (M1)	450	40	6.5	3.5	137	240	365	45.1	279.9
Olympus (M2)	240	40	2.3	2.9	106.8	147	289	68.8	403.1
BudgetSensors (M3)	225	28	3.0	3.2	118.7	215	325	75.2	475.1

The mass densities ρ of the micro-cantilevers, which are made of silicon material, are equal to 2320 $\frac{\text{kg}}{\text{m}^3}$. The effective masses m_e of the micro-cantilevers are calculated using the expression given below [17].

$$m_e = \frac{33}{140}lwt\rho \quad (8)$$

Dynamic acoustic forces are considered to act on the one-side surfaces of the micro-cantilevers [10, 18]. Effective strengths of acoustic forces are determined using Sound Pressure Levels (SPLs) in the range of 30-75 dB [19]. Frequencies of the acoustic forces are generated in the range of 100-800 kHz with a step of 5 kHz in numerical simulation. Magnitudes of the external excitation forces at the first eigenmode for the micro-cantilever M1, M2, and M3 are set to 145.8, 197.3 and 148.8 pN respectively. Magnitudes of the external excitation forces at the second eigenmode for the micro-cantilever M1, M2, and M3 are set to 375.3, 369.6 and 365.2 pN respectively. The total time interval of the numerical simulation is between 19.12 and 19.20 ms. Initial deflections and velocities are set to zero and applied to solve the EOM (Eq. (1)) in Matlab.

3. Results of energy quantities

The acoustic force frequency bands, in which high fluctuations in virials and dissipated powers are observed, are identified for diverse force strengths calculated considering the SPL of 30 dB (Fig. 1). For the micro-cantilever M1, the high-sensitivity frequency range of around 200-270 kHz is determined for the force strengths in the range of 11.4-36.0 pN (Fig. 1(a, b)). The high-sensitivity frequency range of around 440-505 kHz is obtained for the micro-cantilever M2 as the force strength varies in the range of 6.1-19.2 pN (Fig. 1(c, d)). For the micro-cantilever M3, the frequency range with high sensitivity of around 505-570 kHz is acquired for the force strengths in the range of 4.0-12.6 pN (Fig. 1(e, f)). Herein, we would like to remark that high-sensitivity frequency ranges are obtained near the second eigenmode frequencies of the micro-cantilevers. Fluctuations in virials between positive and negative values are evident irrespective of force strengths for the micro-cantilever M1 (Fig. 1(a)). The positive value of virial points out the existence of energy storing process of the resonant micro-cantilever in the presence of acoustic forces [10]. In addition, fluctuations among negative values of virials are observed in high-sensitivity frequency bands as it is illustrated in Fig. 1(c, e). Mechanically oscillating system losses energy owing to the acoustic forces, indicated by positive values of dissipated powers (Fig. 1(b, d, f)). Mechanical energy transferred into the dissipative operation environment is represented by negative value of dissipated power based on virial theorem and energy conservation principle [3]. We have remarked in our earlier study that interaction of the first two flexural modes is prevalent in bimodal operation based on energy quantities [10]. Furthermore, high fluctuations in energy quantities are obtained as the force strength increases in the high-sensitivity frequency regions. There do not exist remarkable distinctions among virials and dissipated powers for diverse force strengths in the low-sensitivity frequency bands.

Variations in energy quantities with respect to acoustic force strengths, determined based on SPLs in the range of 30-75 dB, are illustrated in Fig. 2. The acoustic force at the frequency of 230, 470, 545 kHz is within the high-sensitivity frequency band for the micro-cantilever M1, M2, and M3 respectively. On the other hand, other acoustic force frequencies depicted in Fig. 2 belong to the low-sensitivity frequency bands. The energy quantities exhibit more upward trends with increasing acoustic force strengths in high-sensitivity frequency ranges, when compared with the ones in the low-sensitivity frequency bands. Accordingly, higher response of free oscillations to acoustic forces within high-sensitivity frequency bands can be identified based on energy quantities.

Virials of acoustic force interaction exhibit linear behavior for higher strengths of acoustic forces within high-sensitivity frequency bands (Fig. 2(a, c, e)). Increase in amplitudes owing to influences of external excitation forces and acoustic forces leads to steady increase in virials. Moreover, distinctions among low- and high-sensitivity frequency bands are quite evident based on dissipated power results (Fig. 2(b, d, f)). Energy dissipation owing to interaction with the acoustic forces is increasing as the force strength ascends for high-sensitivity frequency bands.

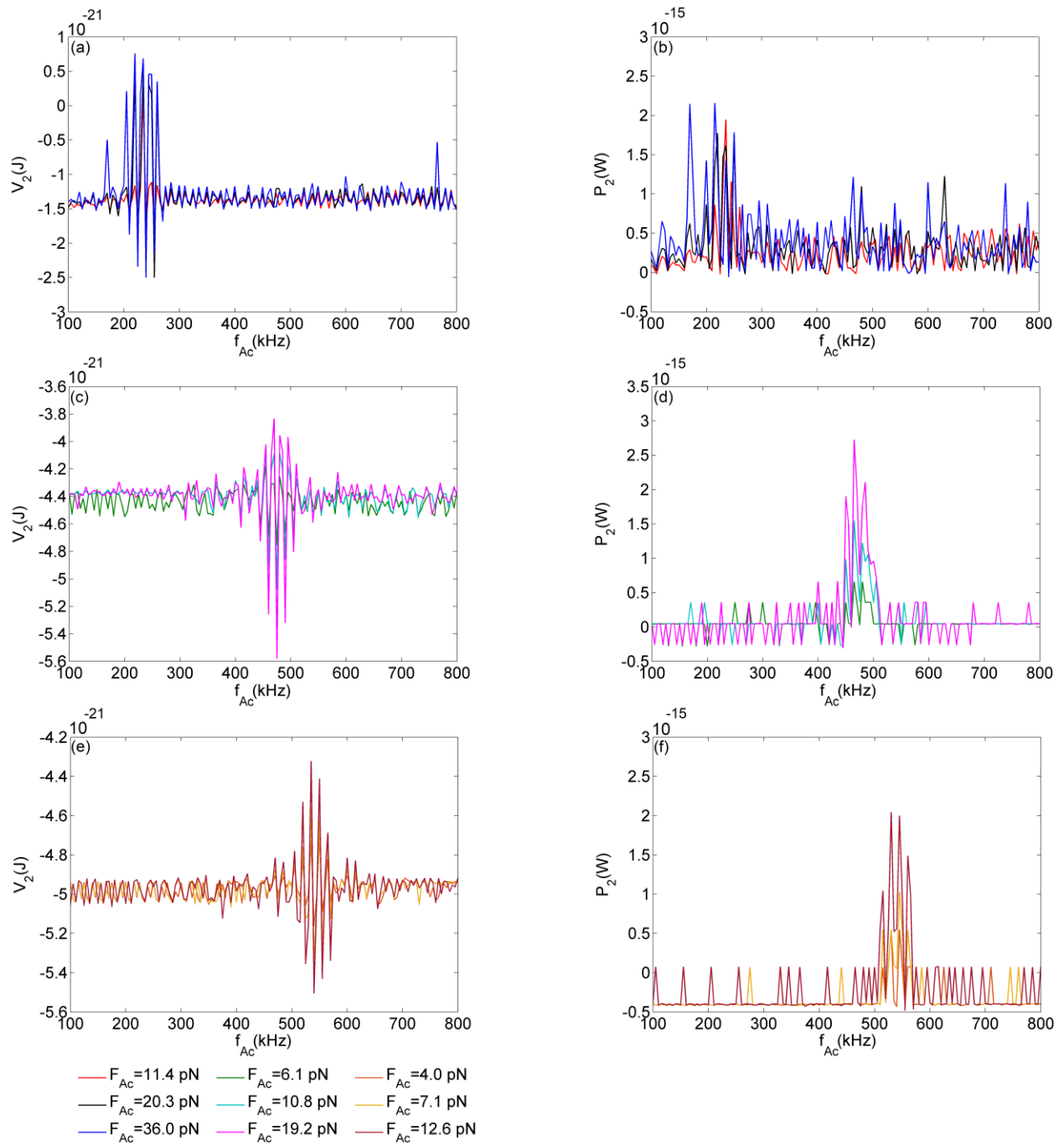


Figure 1. Virials and dissipated powers with respect to frequencies of dynamic acoustic forces with diverse strengths. (a) Virial curves for the micro-cantilever M1. (b) Dissipated power curves for the micro-cantilever M1. (c) Virial curves for the micro-cantilever M2. (d) Dissipated power curves for the micro-cantilever M2. (e) Virial curves for the micro-cantilever M3. (f) Dissipated power curves for the micro-cantilever M3.

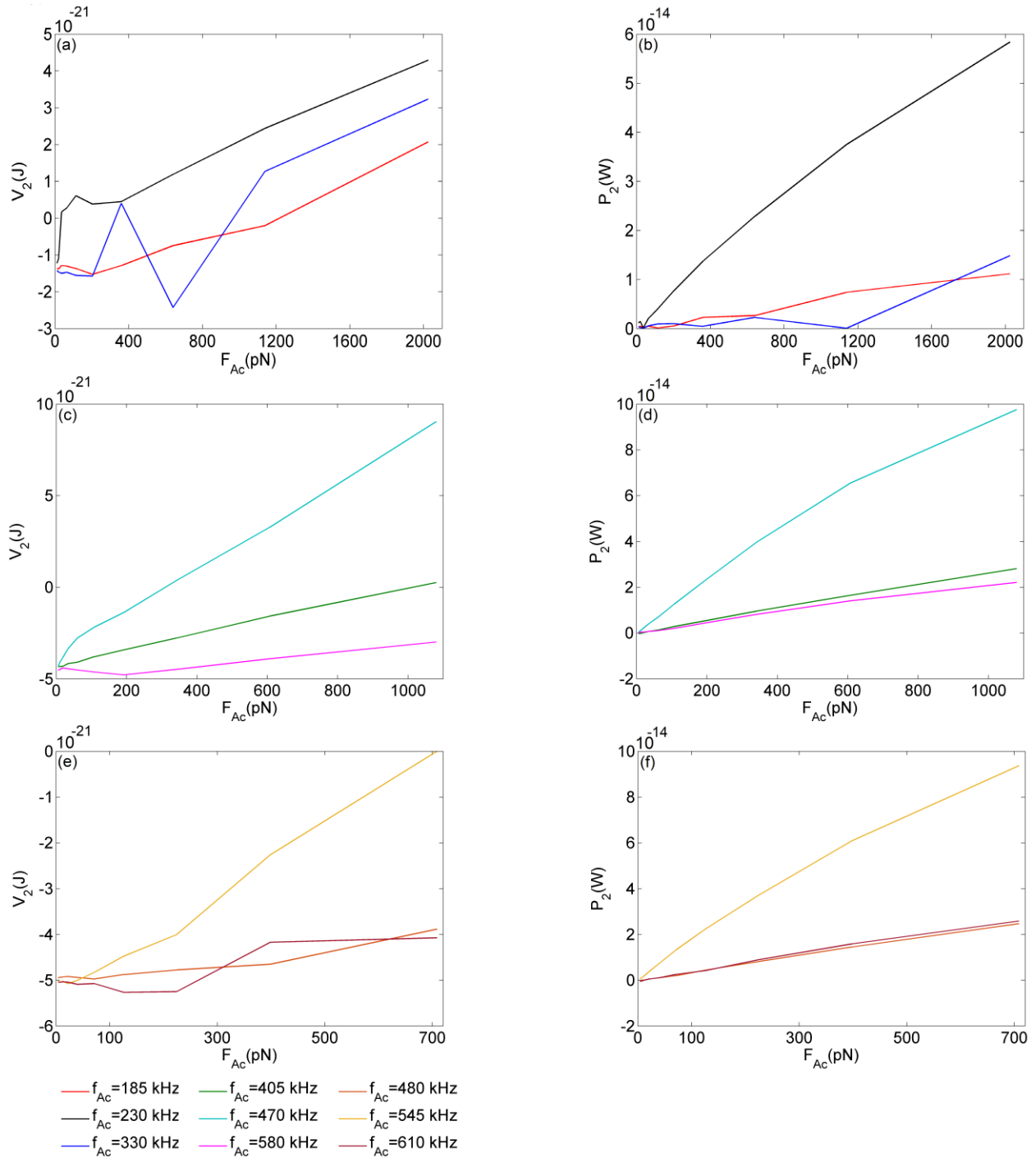


Figure 2. Virials and dissipated powers with respect to strengths of dynamic acoustic forces within low- and high-sensitivity frequency regions. (a) Virial curves for the micro-cantilever M1. (b) Dissipated power curves for the micro-cantilever M1. (c) Virial curves for the micro-cantilever M2. (d) Dissipated power curves for the micro-cantilever M2. (e) Virial curves for the micro-cantilever M3. (f) Dissipated power curves for the micro-cantilever M3.

4. Conclusions

Energy quantities such as virial and dissipated power are determined analytically for acoustic force measurement based on oscillation observables such as amplitude and phase shift. Numerical calculations have proved that a wider high-sensitivity frequency region is achieved by driving the micro-cantilever array under multimodal operations. High-sensitivity frequency bands are determined by identifying remarkable fluctuations in energy quantities. The energy dissipation process occurring in the measurement of dynamic acoustic forces is described in consideration of virial and dissipated power terms. More significantly, boundaries of the high-sensitivity frequency bands in the range of around 200-270 kHz and 440-570 kHz depend considerably on the eigenmode frequencies of the micro-cantilevers. In addition, the simulation results point out that driving the micro-cantilevers at higher eigenmode frequencies enables obtaining higher sensitivities to acoustic forces at higher frequencies. Furthermore, energy quantities exhibit steady and increasing trends for the force strengths in the range of around 400-2000 pN within the high-sensitivity frequency bands. Therefore, variations in energy quantities with respect to frequencies and strengths of acoustic forces can be utilized as complementary sensitivity indicators in the measurement of dynamic acoustic forces.

5. References

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